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GENERALIZED ENGSET FORMULAE FOR QUEUEING SYSTEMS
WITH LIMITED BUFFER SPACE

Summary. We analyze closed Engset service systems with requests having some random space requirement (volume) under assumption that service time of the request depends on its volume. We also assume that each customer (or source of requests) is characterized by his own join distribution of the request volume and service time, and the total requests volume is limited. For such systems we determine the stationary distribution of states of the sources.

Keywords: Engset service system, request volume, buffer space, steady-state distribution of sources states, Erlang system

UOGÓLNIONE WZORY ENGSETA DLA SYSTEMU OBSŁUGI
Z OGRANICZONĄ POJEMNOŚCIĄ PAMIĘCI BUFOROWEJ

Streszczenie. W artykule analizowano zamknięty system obsługi Engseta, w którym każde zapytanie jest charakteryzowane pewną losową objętością, przy założeniu że czas obsługi zależy od objętości zapytania. Zakładamy również, że każde źródło zapytań charakteryzuje się odrębną dystrybucją wspólną czasu obsługi i objętości zapytania oraz objętość sumaryczna zapytań w systemie jest ograniczona. Dla wskazanych systemów wyznacza się rozkład stacjonarny stanów źródeł.

Słowa kluczowe: system obsługi Engseta, objętość zapytania, pamięć buforowa, rozkład stacjonarny stanów źródeł, system Erlanga
1. Introduction

In the paper, we investigate a closed queueing system with blocking of requests, which are served with limited amount of resource of the same type. Such models are used in the theory of telecommunication networks (see ex. [1, 2]). They can be represented as Engset systems [3], if the number of sources generating requests is finite, or as Erlang systems [3, 4], if the requests come to the system from outside.

Multiple variants of above systems under most general assumptions about sources parameters, service time distribution (or the time of using of one or more resource units) and service discipline (or resource shared policy) are investigated in [5–8]. Herewith, as a rule, the common number of shared resource (which models the amount of occupied memory space or the number of busy servers) are assumed to be discrete. This fact allows to simplify the proper investigation.

Generally, the amount of using (by a request) resource, can be an arbitrary positive real number or a vector with real positive components. For example, we can suppose that, in Engset and Erlang models, a request is characterized by some random volume, which is presented by positive real number, and the total volume of requests in the system is limited by some constant value (memory or buffer space volume) $V > 0$.

Some simple queueing models with limited total volume, which are the generalization of the classical Erlang systems, were analyzed in the monographs [9, 10].

In this paper, we analyze the general Engset system under assumption that each source of requests is characterized by its own join distribution function of the request volume and its service time. Herewith, the total volume of requests present in the system is limited by the buffer space volume $V$. For this system, we obtain the distribution of sources states. The results of the paper are the generalization of ones obtained in [5]. The paper is organized as follows. In Section 2, we define the model and introduce the proper notation. In Section 3, we obtain the steady-state distribution of system states. In Section 4, we analyze the case when all sources of the system can be divided on groups of equivalent types of requests. In Section 5, we obtain the formulae for states distribution in Erlang system, as a special case of Engset system. The results of the paper is discussed in Section 6.

2. Model and notation

Consider the generalized closed (Engset) service system with two types of resources. The first one (discrete or continuous) is limited by the value $V > 0$. We shall name it buffer space
volume. The second one (servers) is discrete. At the beginning, we assume that the amount of the second type resource is unlimited.

The requests are generated by \( n \) sources. The \( j \)th source, \( j = 1, n \), generates a request during some random time characterized by the distribution function (DF) \( A_j(t) \). For its service, each request of the source needs some amount \( \zeta_j \) of the first type system resource, which we shall name request space requirement or request volume. We denote by \( \sigma(t) \) the total volume of requests present in the system at time instant \( t \). We assume that \( \zeta_j \) is the non-negative random variable (RV) with DF \( L_j(x) \). At the epoch \( t \) of generating termination, the request of volume \( x \) admits to service, if the total volume \( \sigma(t) \) of other requests present in the system at this time instant is such that \( \sigma(t) = \sigma(t^-) + x \leq V \). In opposite case, the request will be lost with no influence to future system behavior.

The accepted request starts its service. Its service time \( \xi_j \) generally depends on the source number \( j \) and the request volume \( \zeta_j \). We denote by:

\[
P_j(x, t) = P(\zeta_j < x, \xi_j < t), \quad j = 1, n,
\]

the joint DF of RV \( \zeta_j \) and \( \xi_j \). It is evident that \( L_j(x) = F_j(x, \infty) \) and DF of service time of the request of \( j^{th} \) source (\( j \)-request) has the form \( B_j(t) = F_j(\infty, t) \). The proper source starts generation of the next request immediately after service termination of previous one.

We assume that there exist the mean values:

\[
\alpha_{jl} = E\zeta_j = \int_0^\infty t dA_j(t) = \int_0^\infty [1 - A_j(t)] dt, \quad j = 1, n.
\]

\[
\beta_{jl} = E\xi_j = \int_0^\infty t dB_j(t) = \int_0^\infty [1 - B_j(t)] dt, \quad j = 1, n.
\]

If a request has been lost, the proper source immediately starts generation of the next one. The generation time and the request volume don’t depend on a loss of the previous request.

Let \( v_j(t) \) be the state of \( j^{th} \) source at time instant \( t \). Assume that \( v_j(t) = 0 \), if the source is generating a request, and \( v_j(t) = 1 \), if a request of the source is on service at this moment.

At arbitrary time moment, we shall characterize the system under consideration by the state of its sources and by the value of total requests volume at this time. The state of all sources we shall denote by the random vector \( N(t) = (v_1(t), \ldots, v_n(t)) \). Then, the state of the system is characterized by the \( n+1 \) -dimensional vector \( X(t) = (N(t), \sigma(t)) \).

The trajectory of the process \( X(t) \) at arbitrary time instant \( t \) can be described by the states vector \( Q = (r_1, \ldots, r_n, x) \) where \( r_j \) is the realization of the process \( V_j(t) \), \( j = 1, n \), and \( x \) is the realization of the process \( \sigma(t) \). Further, we are interesting with steady-state distribution of the
random vector \( N(t) \). It is clear that the set \( \Omega \) of its states \( R = (r_1, \ldots, r_n) \) is finite. The number \( |\Omega| \) of these states equals \( 2^n \).

Note that, in some papers devoted to Engset systems (see ex. [6, 8]), it were assumed that the resource of second type using for requests service (which can be associated with the set of servers) is limited, so that the set \( G \) of system states doesn’t coincide with \( \Omega \). The structure of the set \( G \subset \Omega \) and its cardinality are determined by the number of units of the second type resource and resource sharing policy (service discipline) [6]. For example, requests are served by finite number of identical servers and \( j - \)request needs \( n_j \) servers for its service. Sometimes, the set of servers can be divided on groups available for requests of certain sources only etc. Now, we shall assume that the resource of second type is unlimited, but the proper corrections will be done below.

For arbitrary vector \( R \) of the process \( N(t) \) states, we introduce the following notation [5]:

\[
R_{j0} = (r_1, \ldots, r_{j-1}, 0, r_{j+1}, \ldots, r_n), \quad R_{ji} = (r_1, \ldots, r_{j-1}, 1, r_{j+1}, \ldots, r_n).
\]

Let \( \xi_j^\ast(t) \) be the time duration from the moment \( t \) to the next epoch of \( j \)th source state changing, \( j = 1, n \). Then, the process of system behavior

\[
(X(t), \xi_j^\ast(t), j = 1, n) = (N(t), \sigma(t), \xi_j^\ast(t), j = 1, n)
\]

can be characterized by functions having the following probability sense:

\[
Z(R, Y, x, t) = P\{N(t) = R, \sigma(t) < x, \xi_j^\ast(t) < y_j, j = 1, n\},
\]

where \( Y = (y_1, \ldots, y_n) \). Introduce also the notation \( P(R, Y, t) = Z(R, Y, V, t) \).

In steady state (when \( t \to \infty \)), we shall use the same notation with omitting the variable \( t \). For example, we shall write \( N \) instead of \( N(t) \) or \( P(R, Y) \) instead of \( P(R, Y, t) \). In particular,

\[
Z(R, Y, x) = \lim_{t \to \infty} Z(R, Y, x, t), \quad P(R, Y) = \lim_{t \to \infty} P(R, Y, t).
\]

3. Steady-state distribution of system states

Let us write out equations for the functions \( P(R, Y) \) using the method of supplementary variables [3]. Indeed, introducing \( n \)-dimensional vectors:

\[
Y_j = (y_1, \ldots, y_{j-1}, \Delta t, y_{j+1}, \ldots, y_n), \quad E_n = (1, \ldots, 1), \quad \infty_n = (\underbrace{\infty, \ldots, \infty}_n), \quad 0_n = (0, \ldots, 0),
\]

we obtain the following difference equations (their number equals \( 2^n \)): 
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\[ P(R, Y - \Delta tE_n, t + \Delta t) = P(R, Y, t) - \sum_{j=1}^{n} P(R, Y_j, t) + \sum_{j=1}^{n} (1 - r_j)A_j(y_j) \times \]
\[ \times \left[ P(R_j, Y_j, t) + P(R_{j0}, Y_j, t) - \int_{0}^{v} Z(R_{j0}, Y_j, V - x, t) \, dL_j(x) \right] + \]
\[ + \sum_{j=1}^{n} r_j \int_{x=0}^{v} Z(R_{j0}, Y_j, V - x, t) \, dF_j(x, y_j) + o(\Delta t). \]

From these equations, we obtain the following steady-state partial differential ones:

\[ \sum_{j=1}^{n} \left[ \frac{\partial P(R, Y)}{\partial y_j} = \frac{\partial P(R, Y)}{\partial y_j} \right]_{y_j=0} + r_j \int_{x=0}^{v} \frac{\partial Z(R_{j0}, Y, V - x)}{\partial y_j} \, dF_j(x, y_j) + (1 - r_j)A_j(y_j) \times \]
\[ \times \left[ \frac{\partial P(R_{j1}, Y)}{\partial y_j} + \frac{\partial P(R_{j0}, Y)}{\partial y_j} \right]_{y_j=0} - \int_{y_j=0}^{v} \frac{\partial Z(R_{j0}, Y, V - x)}{\partial y_j} \, dL_j(x) \right] = 0. \]

It is clear that each of these equations can be replaced by the system including \(n\) equations of the following form:

\[ \frac{\partial P(R, Y)}{\partial y_j} = \frac{\partial P(R, Y)}{\partial y_j} \Bigg|_{y_j=0} - r_j \int_{x=0}^{v} \frac{\partial Z(R_{j0}, Y, V - x)}{\partial y_j} \, dF_j(x, y_j) - (1 - r_j)A_j(y_j) \times \]
\[ \times \left[ \frac{\partial P(R_{j1}, Y)}{\partial y_j} + \frac{\partial P(R_{j0}, Y)}{\partial y_j} \right]_{y_j=0} - \int_{y_j=0}^{v} \frac{\partial Z(R_{j0}, Y, V - x)}{\partial y_j} \, dL_j(x) \right], \quad j = 1, n. \]

For \(r_j = 1\), these equations take the form:

\[ \frac{\partial P(R_{j1}, Y)}{\partial y_j} = \frac{\partial P(R_{j1}, Y)}{\partial y_j} \Bigg|_{y_j=0} - \int_{x=0}^{v} \frac{\partial Z(R_{j0}, Y, V - x)}{\partial y_j} \, dF_j(x, y_j), \quad (1) \]

and, for \(r_j = 1\), we obtain:

\[ \frac{\partial P(R_{j0}, Y)}{\partial y_j} = \frac{\partial P(R_{j0}, Y)}{\partial y_j} \Bigg|_{y_j=0} - A_j(y_j) \left[ \frac{\partial P(R_{j1}, Y)}{\partial y_j} \right]_{y_j=0} + \]
\[ + \int_{y_j=0}^{v} \frac{\partial Z(R_{j0}, Y, V - x)}{\partial y_j} \, dL_j(x) \right]. \quad (2) \]

It is clear that, in steady state, the mean number of \(j\)-requests starting their service during some time interval equals to the mean number of such requests finishing their service during this interval, whereas we obtain the equality:
Then, taking into account the relation (3), the equation (1) can be presented in the form:

\[
\frac{\partial P(R_{j1}, Y)}{\partial y_j} = \int_0^y \frac{\partial Z(R_{j0}, Y, V-x)}{\partial y_j} \, dL_j(x). \tag{3}
\]

Let us introduce the functions:

\[H_j(x, y) = P(\xi_j < x, \xi_j \geq y) = \int_{w=0}^x \int_{u=y}^\infty dF_j(w, u) = L_j(x) - F_j(x, y), \quad j = 1, n.\]

Then, the equation (4) takes the form:

\[
\frac{\partial P(R_{j1}, Y)}{\partial y_j} = \int_{x=0}^y \frac{\partial Z(R_{j0}, Y, V-x)}{\partial y_j} \, d\xi H_j(x, y_j), \tag{4}
\]

whereas we obtain:

\[
P(R_{j1}, Y) = \int_{u=0}^x \int_{x=0}^y \frac{\partial Z(R_{j0}, Y, V-x)}{\partial y_j} \, d\xi H_j(x, y_j). \tag{5}
\]

From the equation (2), we have, taking in consideration the relation (3),

\[
\frac{\partial P(R_{j0}, Y)}{\partial y_j} = \frac{\partial P(R_{j0}, Y)}{\partial y_j} \bigg|_{y_j=0} [1 - A_j(y_j)],
\]

whereas:

\[
P(R_{j0}, Y) = \frac{\partial P(R_{j0}, Y)}{\partial y_j} \bigg|_{y_j=0} \int_0^y [1 - A_j(u)] \, du. \tag{6}
\]

Introduce the notation \( \Phi_j^y(x) = \int_0^y H_j(x, u) \, du \) and the following notation for Stieltjes convolution of the functions \( F_j(x), \ldots, F_n(x) \):

\[F_1 \ast \ldots \ast F_n(x) = \bigg( \ast_{j=1}^n F_j(x) \bigg).\]

By direct substitution, we can easily show that the functions:

\[Z(R, Y, x) = C \prod_{j=1}^n \left\{ \int_0^{y_j} [1 - A_j(u)] \, du \right\}^{1-r_j} \ast_{j=1}^n r_j \Phi_j^{y_j}(x) \]

satisfy the relations (5) and (6), where the constant value \( C \) doesn’t depend on vector’s \((R, x)\) components, as it follows from the equation (3). Hence, we have:
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\[ P(R, Y) = Z(R, Y, V) = C \prod_{j=1}^{n} \left\{ \int_{0}^{y_j} [1 - A_j(u)] du \right\}^{1-r_j} r_j \Phi_j^y(V). \]  

(7)

If all components of the vector \( R \) equal zero (\( R = 0_n \)), we obtain from the relation (7) that:

\[ P(0_n, Y) = C \prod_{j=1}^{n} \int_{0}^{y_j} [1 - A_j(u)] du, \]

whereas we have:

\[ C = P(0_n, Y) \left\{ \prod_{j=1}^{n} \int_{0}^{y_j} [1 - A_j(u)] du \right\}^{-1}. \]

If we substitute the last relations to the relation (7), we obtain:

\[ P(R, Y) = P(0_n, Y) \Phi_j^y(V) \prod_{j=1}^{n} r_j \left\{ \int_{0}^{y_j} [1 - A_j(u)] du \right\}^{-1}. \]

(8)

It is clear that the stationary distribution \( \Pi(R) \) of the system states (or vector of states \( R = (r_1, \ldots, r_n) \)) can be calculated as:

\[ \Pi(R) = P(R, \infty) = \lim_{y_j \to \infty, j=1, n} P(R, Y). \]

Introduce the notation:

\[ D_j(x) = \int_{u=0}^{x} \int_{y=0}^{\infty} y \, dF_j(u, y) = E(\xi_j; \xi_j < x) = E(\xi_j | \xi_j < x)L_j(x). \]

Here the function \( D_j(x) \) is the partial mean value of RV \( \xi_j \) with respect to the event \( \xi_j < x \) [11], \( E(\xi_j | \xi_j < x) \) is the conditional mean value of RV \( \xi_j \) under condition \( \xi_j < x \).

It can be easy shown that \( \lim_{y_j \to \infty} \Phi_j^y(x) = D_j(x) \). Therefore, passing in (8) to limit when \( y_j \to \infty, j=1, n \), we obtain:

\[ \Pi(R) = \Pi(0_n) \left( \prod_{j=1}^{n} D_j(V) \right) \prod_{j=1}^{n} r_j / \alpha_j, \]

(9)

where \( \Pi(0_n) = P(0_n, \infty) \).

It follows from the relation (9) that \( \Pi(R) \) doesn’t depend on the type of functions \( A_j(t) \). It depends on means of the proper RV only. The multiplier \( \Pi(0_n) \) can be obtain from the normalization condition. Generally, summing on all \( R \in \Omega \) has been replaced by summing on the set \( G \) of all system states. Finally, we have:

\[ \Pi(0_n) = \left[ \sum_{R \in G} \left( \prod_{j=1}^{n} D_j(V) \right) \prod_{j=1}^{n} r_j / \alpha_j \right]^{-1}. \]

(10)
4. Engset formulae for the system with equivalent types of requests

In the system under consideration, all sources with the same values \( \alpha_{j_1} \) and the same functions \( D_j(x) \) we shall name equivalent. Their requests are \( j \)-requests or requests of \( j \)th type.

Consider closed service system with \( k \) types of equivalent sources. There are \( n_j \) sources of \( j \)th type, \( j = 1, k \), \( \sum_{j=1}^{k} n_j = n \).

In this case, we shall characterize the states of the system in steady state by the vector \( M = (m_1, \ldots, m_k) \) where \( m_j, j = 1, k \), is the numbers of sources, whose requests are served at this time instant. The set of all possible states of the system (i.e. the set of possible values of \( M \)) we denote by \( K \).

Introduce the notation \( Q_{m_j}(x) = D_j^{(m_j)}(x) \), where \( D_j^{(m_j)}(x) \) is the \( m_j \)-fold Stieltjes convolution of the function \( D_j(x) \). Then, from the relation (9), we derive that the distribution \( \Pi(M) \) of system states takes the form:

\[
\Pi(M) = \Pi(0) \left( \prod_{j=1}^{k} \left( \prod_{j=1}^{n_j} \frac{n_j!}{m_j!} \right) \right)^{-1} \frac{1}{\alpha_{j_1}^{m_j}},
\]

where:

\[
\Pi(0) = \sum_{M \in K} \left( \prod_{j=1}^{k} \left( \prod_{j=1}^{n_j} \frac{n_j!}{m_j!} \right) \right)^{-1} \frac{1}{\alpha_{j_1}^{m_j}}.
\]

Using the notation \( a_j = n_j / \alpha_{j_1} \), we get:

\[
\Pi(M) = \Pi(0) \left( \prod_{j=1}^{k} \left( \prod_{j=1}^{n_j} \frac{n_j!}{m_j!} \right) \right)^{-1} \frac{a_j^{m_j}}{n_j^{m_j}}.
\]

For example, if service time of requests doesn’t depend on their volume, we have \( D_j(x) = \beta_j L_j(x) \) and \( Q_{m_j}(x) = \beta_j S_j(x) \), \( j = 1, k \), where \( S_j(x) = L_j^{(m_j)}(x) \), whereas we obtain:

\[
\Pi(M) = \Pi(0) \left( \prod_{j=1}^{k} \left( \prod_{j=1}^{n_j} \frac{n_j!}{m_j!} \right) \right)^{-1} \frac{a_j \beta_j^{m_j}}{n_j^{m_j}}.
\]

If service time is proportional to the request volume (\( \xi_j = c_j \xi_j \), \( c_j > 0 \), \( j = 1, k \)), we have \( D_j(x) = c_j \int_0^{t} dL_j(t) \). In these cases, the precise calculation of Stieltjes convolutions in the
relation (11) is possible, for example, when request volume $\zeta_j$, $j = 1, k$, has gamma distribution with density:

$$l_j(x) = \frac{x^{\delta_j-1}e^{-fx}}{\Gamma(\delta_j)}, \delta_j > 0, f > 0,$$

where parameter $f$ is the same for all $j$ (see [12] for details).

Now, let all $n$ sources belong to the same type $j = 1$ with the mean of generation time $\alpha_1$ and the function $D(x) = D_1(x)$ and let $p_k$ be the probability that there are $k$ requests in the system in steady state, $k = 0, n$. Then, we have from the relation (11):

$$p_k = p_0 D_1^{(k)}(V) \left(\frac{a}{n}\right)^k, k = 1, n,$$

where:

$$p_0 = \left[1 + \sum_{k=1}^{n} D_1^{(k)}(V) \left(\frac{a}{n}\right)^k\right]^{-1}, a = \frac{n}{\alpha_1}.$$

5. Erlang open system

Assume that we have $n_j$ equivalent sources of $j^{th}$ type, $j = 1, k$, for which the relation $a_j = n_j / a_{j1}$ holds (i.e. the value $a_j$ doesn’t change with the number of sources $n_j$ increasing). Then, the summarized entrance flow from all sources of $j^{th}$ type becomes stationary Poisson with parameter $a_j$, when $n_j \to \infty$, $j = 1, k$ [4]. So, we obtain the open service system with $k$ independent Poisson requests entrance with parameters $a_j$, $j = 1, k$. The requests of $j^{th}$ type are characterized by the joint DF $F_j(x, t)$ of the random volume $\zeta_j$ and service time $\xi_j$. This is an Erlang system with limited (by the value $V$) buffer space volume. The set $K$ of its states depends on the number of discrete resource units and its sharing policy.

Passing in the relation (11) to limit when $n_j \to \infty$, $j = 1, k$, we obtain the following relation for probability of the state $M \in K$:

$$\Pi(M) = \Pi(0_j) \left(\prod_{j=1}^{k} Q_{m_j}(V)\right) \prod_{j=1}^{k} \frac{a_j^{m_j}}{m_j!},$$

(12)
where:

\[
\Pi(0_k) = \left[ \sum_{M \in \mathcal{K}} \left( \prod_{j=1}^{k} Q_{m_j}^{(V)}(V) \right) \prod_{j=1}^{k} \frac{a_{q_j}^{m_j}}{m_j!} \right]^{-1}.
\]

(13)

For example, let us consider Erlang system \( M/G/l/(0, V) \) [12] with limited by \( V \) buffer space volume. Assume that \( j \)-request service needs \( j \) servers and this request will be lost, if the number of free servers is less than \( j \) at the epoch of the request arriving. Requests of \( j^{th} \) type are characterized by joint DF \( F_j(x, t) \) of their volume and service time. Let \( a \) be the rate of summarized entrance flow and \( q_j \) be the probability that arbitrary request is one of \( j^{th} \) type. Then, \( \mathcal{K} \) is the set of such vectors \( M = (m_1, \ldots, m_k) \), for which the inequality:

\[
m_{(n)} = m_1 + \ldots + m_n \leq n
\]

holds. Hence, we obtain:

\[
\Pi(M) = \Pi(0_k) \left( \prod_{j=1}^{k} Q_{m_j}^{(V)}(V) \right) \prod_{j=1}^{k} \frac{(a_{q_j})^{m_j}}{m_j!},
\]

where \( m_{(n)} \leq n \) and

\[
\Pi(0_k) = \left[ \sum_{m_{(n)} \leq n} \left( \prod_{j=1}^{k} Q_{m_j}^{(V)}(V) \right) \prod_{j=1}^{k} \frac{(a_{q_j})^{m_j}}{m_j!} \right]^{-1}.
\]

In particular, if all requests belong the same type characterizing by the joint DF \( F(x, t) \) of the request volume \( \zeta \) and its service time \( \xi \) and each request needs a single server, the steady-state probability \( p_k \) of the presence of \( k \) requests in the system takes the form [12]:

\[
p_k = p_0 \frac{a^k}{k!} D^{(k)}(V), \quad k = 0, n,
\]

where \( D^{(k)}(x) \) is the \( k \)-fold Stieltjes convolution of the function:

\[
D(x) = \int_{y=0}^{\infty} \int_{y=0}^{\infty} y dF(w, y) = E(\xi; \xi < x)
\]

\[
p_0 = \left[ 1 + \sum_{k=1}^{n} \frac{a^k}{k!} D^{(k)}(V) \right]^{-1}.
\]

6. Conclusions

In the paper, we investigate closed Engset systems with requests of random volumes and service time depending on their volumes under assumption that each source of requests is characterized by its own joint distribution of request volume and its service time and the total
requests volume in the system is limited by buffer space volume \( V \). For the system under consideration, we obtain the sources states distribution, which is characterized in the paper by the random vector \( N(t) \).

As it follows from the relations (9) and (10), the steady-state distribution of the vector \( N(t) \) is determined by:
1) the value \( V \) of the first type resource amount (buffer space volume) of the system.
2) the number \( n \) of sources.
3) the mean values \( \alpha_{j1}, \ j = 1, \ldots, n \), of the requests generation time.
4) the functions \( D_j(x) = \int_{u=0}^{x} \int_{y=0}^{\infty} y dF_j(u, y), \ j = 1, \ldots, n \). 
5) the number of units of resource of second type and its sharing policy, i.e. the structure of the set \( G \) of system states.

As a special case, we also analyze the open Erlang system.

The results obtained in the paper can be used for estimating of parameters characterizing requests service in the nodes of computer and communication networks.

**BIBLIOGRAPHY**


Omówienie

W artykule przedstawiono analizę zamkniętego systemu obsługi Engseta (ten system nie zawiera kolejki), w którym zapytania są charakteryzowane pewną losową objętością wpływającą na czas obsługi. Zapytania są wysyłane do systemu z różnych źródeł. Każde źródło charakteryzowano odrębnym wspólnym rozkładem objętości wysyłanych zapytań i ich czasu obsługi. Objętość sumaryczna zapytań obsługiwanych w dowolnej chwili czasu jest ograniczona pewną wielkością stałą, którą traktowano jako pojemność pamięci buforowej systemu.

Dla opisanego systemu wyprowadzono wzory na obliczanie rozkładu jego stanów w trybie stacjonarnym, tj. stacjonarnych prawdopodobieństw łącznych dla wszystkich możliwych kombinacji stanów źródeł. Przeanalizowano również przypadek podziału wszystkich źródeł na klasę równoważnych. Jako przypadek graniczny otrzymano wzory na prawdopodobieństwa stanów dla otwartego systemu Erlanga.

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